DISREGARDING SPECTRAL OVERLAP - A UNIFIED APPROACH FOR DEMOSAICKING, COMPRESSIVE SENSING AND COLOR FILTER ARRAY DESIGN

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ABSTRACT

Modern Color Filter Array (CFA) designs avoid overlap of luminance and chrominance spectra as it is widely believed to cause irreversible crosstalk. In this paper, we present a simple formulation of the demosaicking problem that disregards spectral overlap - and thus the geometry of the spectra - and instead connects the luminance and chrominance resolutions in a more fundamental way to the photosite density of the sensor.

A linear universal demosaicker consisting of space variant filters results from this formulation along with CFA designs that capture more image information than competing designs. This includes the interesting class of random RGB designs that outperform the Bayer CFA with nonlinear demosaickers in terms of PSNR and have a number of additional desirable properties.

Furthermore, we present non-linear enhancements to our universal demosaicker and show them to be either related to compressive sensing or an alternate reconstruction algorithm for it.

***Index Terms*—** Color Filter Array, Compressive Sensing, Universal Demosaick , Robust Demosaick

1. INTRODUCTION

A popular technique of capturing color images with a single sensor is to overlay its photosites with a mosaic of color filters. Perhaps the simplest Color Filter Array (CFA) design is the venerable color stripe CFA that tiles the sensor with alternating stripes of Red, Green and Blue. A more recent and popular design is the Bayer CFA [1].

Interest in CFA design was rekindled by the introduction of frequency domain demosaicking [2] and CFA design techniques [3]. A quick succession of papers optimized CFA design techniques and presented improved CFA instances. The notable objectives include the minimization of pixel density by close packing of circular image spectra into the rectangular spectral support of the sensor [4, 5, 6] , optimization of the aspect ratio of sensor spectral support by controlling the aspect ratio of the photosites [4], minimization of overlap between modulated copies of image spectra [4, 5, 6] , color spaces that minimize chrominance energy[5, 7] , quadrature modulation of image signals [4, 5], maximization of CFA transmittance [5], maximization of demosaicker numerical stability [6], and avoidance of carriers with zero horizontal or vertical frequencies [3].

Packing spectra *without overlap* exacts a heavy toll on resolution as many sensor frequencies go unused. A hypothetical CFA with as many coefficients in its DFT as those in the luminance and chrominance DFTs combined would be optimal. We show how to design such CFAs and their associated demosaicking algorithms, both for spectrally dense and spectrally sparse images. In the process we devise a linear universal demosaicker comprised of space variant filters. Previous work on universal demosaicking includes the application of a local color ratio model [8] and regularization approaches [9] including a variational minimization approach [10]. We are not aware of any previous demosaicking algorithm that outperforms the Bayer CFA demosaicked with modern algorithms in terms of PSNR or other objective metric.

We also show the random RGB to be a universal CFA that is optimal for all ratios of luminance to chrominance bandwidths. This is in contrast to the non-overlapping spectra case wherein the CFA aspect ratio, and color pattern depend on the luminance to chrominance bandwidth ratio [4].

2. FORMULATION OF THE DEMOSAICKING PROBLEM

Consider a discrete image with (N_1, N_2) pixels. Denote the R, G, B color planes of the image by $\boldsymbol{x}_i, i \in \{r, g, b\}$ and those of the CFA that filters it by $\boldsymbol{c}_i, i \in \{r, g, b\}$. Now, a photosite located at $\boldsymbol{n} = (n_1, n_2), 1 \leq n_1 \leq N_1, 1 \leq n_2 \leq N_2$ filters the incident light $\boldsymbol{x}(\boldsymbol{n}) = \begin{bmatrix} x_r(\boldsymbol{n}) & x_g(\boldsymbol{n}) & x_b(\boldsymbol{n}) \end{bmatrix}^T$ through color filter array $\boldsymbol{c}(\boldsymbol{n}) = \begin{bmatrix} c_r(\boldsymbol{n}) & c_g(\boldsymbol{n}) & c_b(\boldsymbol{n}) \end{bmatrix}$ and measures the resulting noise-free, scalar signal $y(\boldsymbol{n})$, where

$$y(\boldsymbol{n}) = \boldsymbol{c}(\boldsymbol{n}) \cdot \boldsymbol{x}(\boldsymbol{n}) \tag{1}$$

Taking (N_1, N_2) point 2D DFT of both sides we have,

$$\boldsymbol{Y} = \sum_{i \in \{r,g,b\}} \boldsymbol{C}_i * \boldsymbol{X}_i \tag{2}$$

where $\boldsymbol{Y}$, $\boldsymbol{C}_i$, $\boldsymbol{X}_i, i \in \{r, g, b\}$ are the 2D DFT of y, $\boldsymbol{c}_i$, $x_i, i \in \{r, g, b\}$ respectively and $*$ represents 2D convolution. In order to cast equation 2 in matrix form, we first define $\tilde{\boldsymbol{Y}}, \tilde{\boldsymbol{X}}_i, \tilde{\boldsymbol{C}}_i, i \in \{r, g, b\}$ as the row-major column vector versions of $\boldsymbol{Y}, \boldsymbol{X}_i, \boldsymbol{C}_i, i \in \{r, g, b\}$ respectively and $\tilde{\boldsymbol{X}} \equiv \begin{bmatrix} \tilde{\boldsymbol{X}}_r \\ \tilde{\boldsymbol{X}}_g \\ \tilde{\boldsymbol{X}}_b \end{bmatrix}$ as the concatenation of $\tilde{\boldsymbol{X}}_r, \tilde{\boldsymbol{X}}_g, \tilde{\boldsymbol{X}}_b$. Furthermore, denote DFT frequencies of the input image by $\boldsymbol{\omega} = (\omega_1, \omega_2)$ and those of the CFA filtered image by $\boldsymbol{\Omega} = (\Omega_1, \Omega_2)$ and their row-major column vector versions as $\tilde{\omega}$ and $\tilde{\Omega}$ respectively. Now equation 2 can be re-written in matrix form as

$$\tilde{\boldsymbol{Y}} = \boldsymbol{A} \cdot \tilde{\boldsymbol{X}} \tag{3}$$

where row $\tilde{\Omega}$ of A is the concatenation of $\tilde{\boldsymbol{D}}_i(\tilde{\Omega}), i \in \{r, g, b\}$,

$$\boldsymbol{A}(\tilde{\Omega}) \equiv \begin{bmatrix} \tilde{\boldsymbol{D}}_r(\tilde{\Omega}) & \tilde{\boldsymbol{D}}_g(\tilde{\Omega}) & \tilde{\boldsymbol{D}}_b(\tilde{\Omega}) \end{bmatrix} \tag{4}$$

and $\tilde{\boldsymbol{D}}_i(\tilde{\Omega}), i \in \{r, g, b\}$ is a row vector obtained by appropriately rearranging the elements of $\tilde{\boldsymbol{C}}_i^T, i \in \{r, g, b\}$ so as to effect the convolution of equation 2. This system of linear equations can be solved to determine $\tilde{\boldsymbol{X}}$ if the rank of $\boldsymbol{A}$ is no less than $|\tilde{\boldsymbol{X}}|$, where $|.|$ denotes cardinality.

$$\tilde{\boldsymbol{X}} = \boldsymbol{A}^{-1} \cdot \tilde{\boldsymbol{Y}} \tag{5}$$

where $\boldsymbol{A}^{-1}$ is the generalized inverse of $\boldsymbol{A}$.

We need only as many sensor elements, $|\tilde{\boldsymbol{Y}}|$, as required to get the rank of matrix $\boldsymbol{A}$ to exceed the number of non-zero DFT coefficients of the input image $\tilde{\boldsymbol{X}}$. Since the rank of matrix $\boldsymbol{A}$ cannot exceed $|\tilde{\boldsymbol{Y}}|$ which itself is one third of the cardinality of $\tilde{\boldsymbol{X}}$ for the case of three basic colors, the above condition can be met if the signal is band-limited, or has sparse spectral support so that two thirds or more elements of $\tilde{\boldsymbol{X}}$ may be taken to be zero.

$\tilde{\boldsymbol{X}}$ can also be recovered if additional a priori information is available, such as low chrominance bandwidth. In this situation, equation 3 has to be augmented with

$$\boldsymbol{Y}' = \boldsymbol{A}' \cdot \tilde{\boldsymbol{X}} \tag{6}$$

that sets the spatial high frequencies of two or more basic colors to be equal. *Spectral overlap has no bearing on the solvability of equations 3, 6.*

3. EFFICIENT CFA DESIGNS

CFA frequencies that contain no modulated image frequencies result in the corresponding row of $\boldsymbol{A}$ being zero and do not increase the rank of $\boldsymbol{A}$, $\mathrm{rk}(\boldsymbol{A})$. Nor do rows of $\boldsymbol{A}$ that are linearly dependent on other rows, which is a shortcoming of the Bayer CFA. *Hence we reverse the conventional wisdom of designing CFAs to avoid spectral overlap while tolerating unused frequencies to one of filling all CFA frequencies with modulated image signal while tolerating spectral overlap in the process.*

A CFA composed of a sufficiently large number of carriers evenly distributed over its frequency support can be designed to have $\mathrm{rk}(\boldsymbol{A}) = |\tilde{\boldsymbol{Y}}|$. Such a CFA consists of a periodic panchromatic pattern. Similarly, a CFA comprising of randomly chosen carrier frequencies, will also have $\mathrm{rk}(\boldsymbol{A})$ close to $|\tilde{\boldsymbol{Y}}|$. Such a CFA comprises a large set of random panchromatic colors.

An RGB CFA with a large $\mathrm{rk}(\boldsymbol{A})$ is feasible, though it is not clear how to design it in the frequency domain. This problem is, however, trivial in the spatial domain - any CFA with randomly arranged RGB filters consists of a large number of carriers of varying amplitudes and is almost certain to have $\mathrm{rk}(\boldsymbol{A})$ close to $|\tilde{\boldsymbol{Y}}|$.

3.1. Full Chrominance Bandwidth Reconstruction

Full chrominance reconstruction can be done by enumerating the DFT coefficients of R, G, B in $\tilde{\boldsymbol{X}}$ that lie in the circular spectral support of the optical image and selecting a CFA such that $|\tilde{\boldsymbol{X}}| \approx \mathrm{rk}(\boldsymbol{A}) \approx |\tilde{\boldsymbol{Y}}|$. The augmented equation 6 is not used as no additional information can be injected. We have experimentally verified $\mathrm{rk}(\boldsymbol{A})$ to be practically equal to $|\tilde{\boldsymbol{Y}}|$, for a random RGB CFA and over 300dB reconstruction using the said CFA with $\pi/4 = 0.78$ times as many photosites as that of the equivalent color stripe design. This is the theoretical lower bound on the CFA photosite count and is a significant improvement over the optimal no-spectral-overlap solution of [4] which needs 0.91 times as many photosites as the the color stripe CFA and has filters of many unique panchromatic colors.

Furthermore, the color stripe CFA has rectangular photosites with $3:1$ aspect ratio that works at the Nyquist limit. This design is prone to aliasing from real world optical low pass filters (OLPFs) that leak frequencies beyond their cut offs. The random RGB CFA, on the other hand, *can have photosites of any geometry* as long as it is finer than the Nyquist rate in all directions. When designed with square photosites, their pitch is $\sqrt{3\pi/4} = 1.53$ times as fine as the Nyquist pitch, which greatly reduces aliasing. Frequencies in the range $[1, 1.53]$ times the OLPF cutoff appear as crosstalk which is randomized into less objectionable noise with dense spectral support that is amenable to noise reduction. This, in turn, allows less aggressive OLPFs to be employed with the concomitant improvement in high frequency SNR.

3.2. Robust Reconstruction

The formulation of equations 3, 6 with $|\tilde{\boldsymbol{X}}| \approx \mathrm{rk}(\boldsymbol{A}) \approx |\tilde{\boldsymbol{Y}}|$ works well for clean images. In our experiments we have achieved essentially perfect reconstruction (PSNR $>$ 300dB) for 8 bit images with full chrominance bandwidth.

For noisy images, CFAs with large clumps of filters of the same color lead to numerical instability when reconstructing the missing colors. CFAs with well mixed colors such as the blue noise random RGB CFAs of [11] overcome this problem. We believe that the explanation for the good performance of blue noise random RGB CFAs is different from the spectral separation of color signals suggested in [11].

Another technique to make reconstruction more robust to noise is to slightly over-determine the system of equations 3, 6. Pseudoinverse algorithms that minimize MSE, or similar error metrics, in effect find consensus solutions that cancel out large errors. This benefit is quite dramatic, and far outweighs the increase in noise from the resulting reduction is photosite area.

A system with 20% more equalities than unknowns is more robust than the Bayer CFA. If the full chrominance bandwidth sensor of section 3.1 is made over determined, it loses some of its resolution lead over the color stripe sensor but gains resistance to noise and aliasing.

3.3. Reduced Chrominance Bandwidth Reconstruction

Reduced chrominance bandwidth reconstruction allows higher luminance resolution to be extracted from a given sensor than would otherwise be possible. This is done by first enumerating the R, G, B DFT coefficients in $\tilde{\boldsymbol{X}}$ that lie in the circular spectral support of the optical image and setting up equation 3. Next a linear color transform is defined that minimizes chrominance energy. This color transform is applied to the DFT coefficients of $\tilde{\boldsymbol{X}}$ and its bandwidth is limited to the required value by appropriately setting up equation 6. Finally the system of linear equations 3, 6 is solved for $\tilde{\boldsymbol{X}}$. This leads us to the following observation:

Observation 1 *Demosaicking algorithms are not constrained to use any particular color space by the CFA. Color space can be adaptively selected for each image or part of an image so as to optimize reconstruction. This is usually done by using chrominance signals with minimal high frequency energy.*

Corollary 1 *Degrees of freedom of CFA design should not be expended on minimizing chrominance energy, as is commonly done, but should instead be saved for the remaining objectives such as* $\mathrm{rk}(\boldsymbol{A})$*, sensitivity, dynamic range and noise.*

The chrominance energy outside the specified chrominance bandwidth appears as crosstalk which is randomized, if a random CFA is used, and rendered susceptible to noise reduction. As in the case of noise, randomization of crosstalk is stable only if the system of linear equations is over determined. As in section 3.2 a system with 20% more equalities than unknowns works very well.

For the popular case of chrominance bandwidth that is half of luminance bandwidth, a random RGB CFA demosaicked by a linear space variant filter, outlined in section 4.1, in the color space of [12] outperformed the Bayer CFA demosaicked with high quality non-linear algorithms. For experimental details see section 6.

This leads us to the following observation:

Observation 2 *The random RGB CFA optimally multiplexes color component signals regardless of their bandwidths and spectral geometries, and hence can be regarded as an universal CFA.*

4. SPATIAL DOMAIN FORMULATION

The result of section 2 can be expressed in the spatial domain by taking inverse DFT of both sides of equation 3, 6. However, it's simpler to develop it again from equation 1. Let the row-major column vector versions of y and the color planes of x, c be $\tilde{\boldsymbol{y}}, \tilde{\boldsymbol{x}}_i, \tilde{\boldsymbol{c}}_i, i \in \{r, g, b\}$. Let $\tilde{\boldsymbol{x}} \equiv \begin{bmatrix} \tilde{\boldsymbol{x}}_r \\ \tilde{\boldsymbol{x}}_g \\ \tilde{\boldsymbol{x}}_b \end{bmatrix}$ be a column vector formed by concatenating $\tilde{\boldsymbol{x}}_i, i \in \{r, g, b\}$ and $\tilde{\boldsymbol{c}} = \begin{bmatrix} \tilde{\boldsymbol{c}}_r & \tilde{\boldsymbol{c}}_g & \tilde{\boldsymbol{c}}_b \end{bmatrix}$ be a matrix formed by concatenating of $\tilde{\boldsymbol{c}}_i, i \in \{r, g, b\}$. Equation 1 can now be re-written as,

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{c}} \cdot \tilde{\boldsymbol{x}} \tag{7}$$

Next, consider a N_1, N_2 point 2D DFT matrix F whose row $F(\tilde{\omega})$ determines the DFT coefficient of frequency $\tilde{\omega}$ of color i when multiplied with $\tilde{\boldsymbol{x}}_i, i \in \{r, g, b\}$. Also consider a color transform with k components $\alpha^{(j)}, 1 \leq j \leq k$ that are linear combinations of r, g, b thus defined:

$$\alpha^{(j)} = \alpha_r^{(j)} r + \alpha_g^{(j)} g + \alpha_b^{(j)} b \tag{8}$$

where k is usually 3.

Next, define $G^{(j)} = \begin{bmatrix} \alpha_r^{(j)} F & \alpha_g^{(j)} F & \alpha_b^{(j)} F \end{bmatrix}$ by scaling and concatenating 3 copies of F so that $G^{(j)} \cdot \tilde{\boldsymbol{x}}$ is the DFT of color component $\alpha^{(j)}$. Construct a matrix $\boldsymbol{s}$ consisting of rows of $G^{(j)}, 1 \leq j \leq k$ that represent the DFT coefficients of color components to be set to zero. We augment the system of equations 7 by replacing $\tilde{\boldsymbol{c}}$ with $B = \begin{bmatrix} \tilde{\boldsymbol{c}} \\ \boldsymbol{s} \end{bmatrix}$ and $\tilde{\boldsymbol{y}}$ with $\boldsymbol{y}' = \begin{bmatrix} \tilde{\boldsymbol{y}} \\ 0_p \end{bmatrix}$, where 0_p is a vector of p zeros and p is the number of rows of $\boldsymbol{s}$. This results in the solution

$$\tilde{\boldsymbol{x}} = B^{-1} \cdot \boldsymbol{y}' \tag{9}$$

where B^{-1} is the generalized inverse of B.

4.1. Demosaicking as a space variant filter

From equation 9 we find that a basic color value at a pixel location can be computed as a weighted sum of elements of $\boldsymbol{y}'$. Since the only non-zero values of $\boldsymbol{y}'$ are in its sub-vector $\tilde{\boldsymbol{y}}$, this reduces to a space variant filter.

The space variant filter obtained from equation 9 has a large kernel the size of $|\tilde{\boldsymbol{y}}|$. In practical systems, this can be greatly reduced by windowing the filter kernel. Alternately, equation 9 can be set up for a small block around each pixel and only the kernel for this pixel solved for. The latter method not only generates small kernels, but does so with greatly reduced computation.

Another practical consideration is the space required to store the convolution kernels. This can be addressed by using a periodic CFA formed by tiling the sensor with a pattern block, so that the number of convolution kernels is reduced to the block size. Such tiling preserves the random character of a random CFA as long as the block size is not too small. In our experiments, we have found 15×15 to work well for both the kernel size and the pattern block size.

5. NON LINEAR RECONSTRUCTION ENHANCEMENTS

While the space variant filter is adequate to outperform the Bayer CFA, we can improve demosaicking performance by borrowing ideas from modern Bayer demosaickers as well as the field of Compressive Sensing.

5.1. Local bandwidth shaping

Most modern Bayer demosaickers use adaptive directional interpolation that rely on the local image spectra energy being compacted along one direction. This direction is perpendicular to an edge in its vicinity. By assuming the image spectral support to be an ellipse, instead of a circle, we can cut down the number of unknowns $|\tilde{\boldsymbol{X}}|$. We can use the resulting budget surplus of equations to increase luminance and/or chrominance resolution and/or resistance to noise and crosstalk.

An implementation with d directions applies d space variant filters, each tuned for its direction, to reconstruct d versions of the image and picks the appropriate one at each pixel. The algorithm for direction selection borrows from demosaicking techniques.

5.2. Leveraging image spectral sparsity

Transform coefficient sparsity is perhaps the most fruitful image property that a reconstruction algorithm can exploit. This is especially true if an appropriate wavelet transform is used. The key challenge here is to identify the non-zero transform coefficients. Recent work in compressive sensing suggests Basis Pursuit to be the optimal polynomial time algorithm for the problem. A popular alternative is Orthogonal Matching Pursuit (OMP) [13] and its extension Simultaneous Orthogonal Matching Pursuit (SOMP) optimized for color images. [14].

We are studying an enhancement to OMP and SOMP that bootstraps by reconstructing a low bandwidth version of the image using the space variant filter of section 4.1, and then alternates between zeroing out small transform coefficients, followed by increasing the reconstruction bandwidth and solving the resulting system of equations until a stopping criterion is met.

6. EXPERIMENTAL RESULTS

We empirically studied the Blue Noise random RGB CFA of [11] tiled with block size of 15×15 in a Matlab simulation. We compared this against the Bayer CFA.

The random CFA images were demosaicked with chrominance bandwidth set to half the luminance bandwidth in the color space of [12]. The Bayer CFA images were demosaicked with the standard POCS algorithm of [15].

Commercial systems use OLPFs to limit optical image bandwidth to under 83% of the Nyquist frequency in order to reduce crosstalk. We achieved roughly the same effect by low pass filtering the input image to the same bandwidth with a brick wall filter. The reconstructed images were compared with the prefiltered input image and their PSNR was noted.

We elected to test on the challenging and realistic IMAX McMaster image set [17], instead of the frequently criticized Kodak image set. The IMAX McMaster image set is not released yet, but two images were made available to us. In addition to the linear demosaicker, implemented as a space variant filter, we tested adaptive directional enhancements with 2 and 4 directions using elliptical spectral support with aspect ratios of 0.82 and 0.7 respectively. For the 2 direction test we prefiltered the input image to 90% of the Nyquist limit, and sensed directions using two criteria: the homogeneity algorithm of [16] and perfectly by consulting the original image. For the 4 direction test we did not prefilter the input image and directions were sensed perfectly by consulting the original image. While unimplementable, the perfect direction tests indicate the improvement possible from good direction sensing algorithms. The results are given in table 1.

Image	Bayer 83% res.	non-dir. 83% res.	2 dir ahd 90% res	2 dir perf. 90% res	4 dir perf. 100% res
1a	25.4	27.2	26.8	29.6	30.0
9b	27.7	28.0	30.6	33.0	33.7

Table 1. PSNR (dB) of Bayer+POCS, random RGB demosaicked with non-directional linear demosaicking, adaptive homogeneity directed 2 directional, and perfectly sensed directional with 2, 4 directions.

We also conducted robustness experiments by simulating sensor noise. The image reconstructed from the random RGB CFA suffered slightly lower increase in MSE due to noise as that from the Bayer CFA but the panchromatic design of [4] performed best suffering less than half as much increase in MSE.

7. CONCLUSION

In the long history of one dimensional signal encoding, spectral overlap has generally been avoided as it complicates processing without providing a concomitant benefit. Spread Spectrum is a prominent exception wherein randomization is used for security and to handle effects of RF channel degradation.

The practice of avoiding spectral overlap has made its way into the young field of CFA design. It is our contention that this is an oversight since the spectra of 2D signals have a notion of shape that makes packing them a fundamentally different problem than the 1D case. One way of overcoming this challenge of 2D signals is to tolerate spectral overlap.

In this paper we generalize the existing frequency domain CFA design solutions so that demultiplexing of modulated signals is reduced to solving a system of linear equations and spectral overlap is tolerated. Furthermore, windowing the resulting space variant filter reduces the computational burden to the point that commercial still cameras should be able to demosaick several frames per second, and current graphics cards should be able to demosaick 4k digital cinema raw data in real time.

Our framework also predicts very high spectral efficiency for the random RGB CFA which we have empirically verified. In our experiments the random RGB CFA outperformed the Bayer CFA demosaicked with the standard non-linear state of the art algorithm both in the noiseless and noisy cases and reconstructed more visually pleasing images.

We also present non linear enhancements to our universal demosaicking algorithm that leverage more sophisticated spectral models such as local geometry and sparsity, thereby unifying our work with compressive sensing.

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